QATORLARNI ANIQ INTEGRALLARNI HISOBLASHGA TADBIQI

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Annotatsiya:
Fzika va texnika fanlaridagi amaliy masalalarni yechishda masalaning yechimi ko'pincha "integrali olinmaydigan" funksiyalarni integrallashga keladi. Ushbu maqolada qatorlarning aniq integrallarni hisoblash tatbiqidan foydalangan holda bunday masalalarda aniq integrallarni hisoblash usullari masalalarni yechish orqali bayon qilingan.

Kalit so'zlar: integral, qator, masala, o'zgaruvchi miqdor, tekis yaqinlashish.

Ma'lumki, texnikada uchraydigan ko'plab masalalar, masalan yuzalarni hisoblash ba'zi "integrali olinmaydigan" turdagi integrallarga keladi. Lekin integral osti funksiyasi o'z aniqlanish sohasida uzluksizligi tufayli bunday integrallarni hisoblash mumkin, faqat bunday integrallarda biz bilgan integrallash usullaridan emas, balkim "boshqacharoq" yo'l tutiladi.

1-misol. $\int_0^1 \frac{\sin x}{x} dx$ integralni hisoblang.

Yechish: Bu yerda integral ostidagi funksiya $x \neq 0$ qiymatlarda aniqlangan. integralni hisoblash uchun Nyuton-Leybens formulasini ishlatish mumkin emas. Chunki integral osti funksiyaning boshlang'ich funksiyasini elementar funksiyalar ko'rinishida tasvirlay olmaymiz. Shuning uchun bu integralni taqribiy hisoblash uchun qator tushunchasidan foydalanamiz, integral ostidagi funksiya qatorga yoyamiz.

$$\frac{\sin x}{x} = 1 - \frac{x^3}{3!} + \frac{x^4}{5!} + \frac{x^6}{7!} + \ldots$$

Bu yerda qatorni tekis yaqinlashuvchiligi ekanligidan foydalangan holda, hadma-had integrallab

$$\int_0^1 \frac{\sin x}{x} dx = \int_0^1 dx - \frac{1}{3!}\int_0^1 x^2 dx + \frac{1}{5!}\int_0^1 x^4 dx - \ldots = x\Big|_0^1 - \frac{1}{3!}\cdot\frac{x^3}{3}x\Big|_0^1 + \frac{1}{5!}\cdot\frac{x^5}{5}x\Big|_0^1 - \ldots =$$

$$= 1 - \frac{1}{3!\cdot 3} + \frac{1}{5!\cdot 5} - \ldots \approx 0{,}94611,$$ hosil qilamiz.

Qatorni ishoralari almashinuvchi (Leybnits alomatini qanoatlantiradi) bo'lgani uchun uning hadlarini moduli monoton kamayadi, uchta hadi bilan chegaralanib $\frac{1}{7!\cdot 7} = \frac{1}{35280} < 0{,}00003$ xatolikka ega bo'lamiz. Hadlar sonini o'zimiz istagan qiymatda olib xatoliklarni kamaytirish mumkin.

2-misol. $\int_0^x \frac{dx}{1+x^2}$ integralni hisoblang.

Yechish: Integral osti funksiyasi $\frac{1}{1+x^2}$ ni qatorga yoyamiz:

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \ldots$$

O'ng tomondagi qator, (-1, 1) intervalda tekis yaqinlashadi, shu sababli mazkur intervalda integrallasak:

$\int_0^x \frac{dx}{1+x^2} = \frac{x}{1} - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$ hosil bo'ladi. Lekin

$\int_0^x \frac{dx}{1+x^2} = arctgx$. Shuning uchun

$$arctgx = \frac{x}{1} - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

Mukammalroq tekshirishlar, garchi yuqoridagi qator x=1 da uzoqlashuvchi bo'lsa ham, x - ning $-1 < x \le 1$ tengsizliklarni qanoatsizlantiruvchi qiymatlari uchun keyingi tenglik kuchga ega bo'lishi ko'rsatadi. Bu tenglikda x=1 faraz qilib, π uchun Leybensning mashhur qatorini hosil qilamiz:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

Bu qator sekin yaqinlashuvchi bo'lganidan π - ni hisoblash uchun o'ng'aysizdir.
Trigonometriyadan ma'lum bo'lgan:

$$\frac{\pi}{4} = 4arctg\frac{1}{5} - arctg\frac{1}{239}$$

formuladan foydalanib quyidagilarni hosil qilamiz

$$\frac{\pi}{4} = \left[\frac{1}{5} - \frac{1}{3\cdot 5^3} + \frac{1}{5\cdot 5^5} - \frac{1}{7\cdot 5^7} + \ldots\right] - \left[\frac{1}{239} - \frac{1}{3\cdot 239^3} + \frac{1}{5\cdot 239^5} - \frac{1}{7\cdot 239^7} + \ldots\right]$$

3-misol. Ehtimollar nazariyasida $\frac{1}{\sqrt{2\pi}}\int_0^2 e^{-\frac{x^2}{2}}dx$ integral juda katta ahamiyatga ega. Uni hisoblang.

Yechish: Bu integralni hisoblash uchun

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \ldots + (-1)^n\frac{x^n}{n!} + \ldots$$

qatorda x ni $\frac{x^2}{2}$ ga almashtiramiz

$$e^{-\frac{x^2}{2}} = 1 - \frac{x^2}{2} + \left(\frac{x^2}{2}\right)^2 \frac{1}{2!} - \left(\frac{x^2}{2}\right)\frac{1}{3!} + \dots$$

Dalamber alomatidan bu qator [0, x] kesmada tekis yaqinlashuvchi. Shuning uchun

$$\frac{1}{\sqrt{2\pi}}\int_0^x e^{-\frac{x^2}{2}}dx = \frac{1}{2\pi}\int_0^1\left(1 - \frac{x^2}{2} + \frac{x^4}{2!\cdot 2^2} - \dots + (-1)^n\frac{x^{2n}}{n!2^n} + \dots\right)dx =$$

$$= \frac{1}{\sqrt{2\pi}}\left(x - \frac{x^3}{2\cdot 3} + \frac{x^5}{2!\cdot 2^2\cdot 5} - \dots + (-1)^n\frac{x^{2n+1}}{n!2^n(2n+1)} + \dots\right)$$

n- ga ma’lim qiymatlar berib integralni taqribiy qiymatlarini hisoblash mumkin.

Xulosa o’rnida aytish lozimki, integrallarni hisoblashda bu va bu kabi masalalar juda ko’plab uchraydi. Shuning uchun bunday masalalarni yechish usullarini o’rganish[1-16] katta ahamiyatga ega.

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