UMUMIY O'RTA TA'LIM MAKTABLARIDA ANIQMAS INTEGRAL MAVZUSINI O'QITISHNING USLUBLARINI TAKOMILLASHTIRISH

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ANNOTATSIYA
Umumiy o'rta ta'lim maktablaridada aniqmas integral mavzusini o'qitishning ba'zi uslub va metodlariga bo'lgan talabdan kelib chiqib, ushbu maqolada Umumiy o'rta ta'lim maktablarida ba'zi bir aniqmas integrallarga doir misollar yechimlari bayon qilingan.

Kalit so'zlar: maxsus nuqtalar, funksiya aniqmas integrali, murakkab funksiya.

KIRISH
Ma'lumki, matematika fanini o'qitishda ilg'or va zamonaviy usullardan foydalanish, yangi informatsion-pedagogik texnologiyalarni tadbiq qilish muhim ahamiyatga ega. Ta'kidlash joizki, yangi pedagogik texnologiya ta'limning ma'lum maqsadga yo'naltirilgan shakli, usuli va vositalarining maxsulidir. Kuzatuvlar shuni ko'rsatadiki, aksariyat hollarda o'qituvchi dars jarayonida faqat o'zi ishlaydi, o'quvchilar esa kuzatuvchi bo'lib qolaveradilar. Ta'limning bunday ko'rinishi o'quvchilarning aqliy tafakkurini o'stirmaydi, faolligini oshirmaydi, ta'lim jarayonidagi ijodiy faoliyatni so'ndiradi.
Ta'limda pedagogik texnologiyalarning asosiy maqsadi o'qitish tizimida o'quvchini dars jarayonining markaziga olib kelish, o'quvchilarni o'quv materialini shunchaki yod olishlaridan, avtomatik tarzda takrorlashlaridan uzoqlashtirib, mustaqil va ijodiy faoliyatini rivojlantirish, darsning faol ishtirokchisiga aylantirishdir. Shundagina o'quvchilar muhim hayotiy yutuq va muammolar, o'tiladigan mavzularning amaliyotga tatbiqi bo'yicha o'z fikriga ega bo'ladi, o'z nuqtai nazarini asoslab bera oladi.
Matematika fanini o'qitishda. O'qituvchi interfaol metodlardan mavzuga muvofiqini tanlay bilishi muhim hisoblanadi. O'qituvchi interfaol metodlardan avvalo oddiydan murakkabga o'tish nazariyasiga amal qilgan holda foydalanmog'i lozim. Qo'llaniladigan interfaol metodlar keng yoritilgan. Bu metodlarning yutuq va kamchiliklari sanab o'tilgan. Metodlarni qo'llash bo'yicha namunalar berilgan.

Aniqmas integral klassik ta'rifi.
Aniqmas integralning klassik ta'rifi Agar (a,b) da f(x) funksiya biror F(x) funksiyaning hosilasiga teng, ya'ni (a,b) intervaldan olingan ixtiyoriy x uchun f(x) funksiyaning boshlang'ch funksiyasi deyiladi. Ravshanki, agar biror oraliqda F(x) funksiya f(x) ning boshlang'ich funksiyasi bo'lsa, u holda ixtiyoriy o'zgarmas C son uchun F(x)+C funksiya ham f(x) ning boshlang'ich funksiyasi bo'ladi, chunki(F(x)+C)'=F'(x)=f(x).Geometrik nuqtai nazardan bu teorema f(x) funksiyaning aniqmas integrali y=F(x)+C bir parametrli (1-rasm) egri chiziqlar oilasini ifodalaydi (C-parametr). Bu egri chiziqlar oilasi quyidagi xossaga ega: egri chiziqlarga abssissasi x=x0 bo'lgan nuqtasida o'tkazilgan urinmalar bir-biriga parallel bo'ladi.

Integrallash usullari
Aniqmas integralda o'zgaruvchini almashtirish.
1-misol. $\int \sqrt{a^2 - x^2}\, dx$aniqmas integralni toping.

Yechish. $x = asint$desak, $dx = acostdt$ bo‘ladi va aniqmas integral ushbu ko‘rinishni oladi:

$$\int \sqrt{a^2 - x^2}dx = \int \sqrt{a^2 - a^2sin^2t} \cdot asoctdt =$$
$$= \int \sqrt{a^2cos^2t} \cdot axostdt = a^2 \int \frac{1 + cos2t}{2}dt =$$
$$= \frac{a^2}{2}\int dt + \frac{a^2}{2}\int cos2tdt = \frac{a^2}{2}t + \frac{a^2}{2}\int costdt =$$
$$= \frac{a^2}{2}t + \frac{a^2}{2}\cdot\frac{1}{2}\int cos2td(2t) = \frac{a^2}{2}t + \frac{a^2}{2}sin2t + C.$$

*E*ndi

$$t = arcsin\frac{x}{a}vasin2t = 2sintcost =$$
$$= 2sint\sqrt{1 - sin^2t} = 2\cdot\frac{x}{a}\sqrt{1 - \frac{x^2}{a^2}}$$

tengliklardan foydalanib oldingi o‘zgaruvchi x ga qaytamiz:

$$\int \sqrt{a^2 - x^2}dx = \frac{a^2}{2}arcsin\frac{x}{a} + \frac{a^2}{4}\cdot 2\cdot\frac{x}{a}\sqrt{1 - \frac{x^2}{a^2}} + C =$$
$$= \frac{a^2}{2}arcsin\frac{x}{a} + \frac{x}{2}\sqrt{a^2 - x^2} + C.$$

2-misol. Aniqmas integralni toping:

$$\int \frac{\sqrt{x^2 + a^2}}{x^2}dx$$

Yechish. $x = atgt$ deb belgilasak, $dx = \frac{adt}{cos^2t}$bo‘ladi. Buni xisobga olib aniqmas integralni topamiz:

$$\int \frac{\sqrt{x^2 + a^2}}{x^2}dx = \int \frac{\sqrt{a^2tg^2t + a^2}}{a^2tg^2t}\cdot\frac{adt}{cos^2t} =$$
$$= \int \frac{\sqrt{1 + tg^2t}}{sin^2t}dt = \int \frac{dt}{sin^2t \cdot cost} =$$
$$= \int \frac{cos^2t + sin^2t}{sin^2t \cdot cost}dt = \int \frac{cost}{sin^2t}dt + \int \frac{dt}{cost} =$$
$$= \int \frac{d(sint)}{sin^2t} + \int \frac{dt}{cost} = -\frac{1}{sint} + \ln\left|\frac{1}{cost} + tgt\right| + C =$$
$$= -\frac{\sqrt{1 + \frac{x^2}{a^2}}}{\frac{x}{a}} + ln\left|\sqrt{1 + \frac{x^2}{a^2}} + \frac{x}{a}\right| + C =$$
$$= -\frac{\sqrt{a^2 + x^2}}{x} + ln\left|\frac{\sqrt{a^2 + x^2} + x}{a}\right| + C.$$

3-misol. Aniqmas integralni toping:

$$\int \frac{dx}{x\sqrt{2x-9}}.$$

Yechish. Ildiz ostidagi ifodani t^2 bilan belgilasak,

$$\int \frac{dx}{x\sqrt{2x-9}} = \left\{\begin{matrix} 2x-9=t^2, \quad t=\sqrt{2x-9}; \\ x=\frac{1}{2}(t^2-9); \quad dx=tdt \end{matrix}\right\} =$$

$$= \int \frac{tdt}{\frac{1}{2}(t^2+9)\cdot t} = 2\int \frac{dt}{9+t^2} = \frac{2}{3}arctg\frac{1}{3}+C = \frac{2}{3}arctg\,\frac{\sqrt{2x-9}}{3}+C.$$

4. misol. Aniqmas integralni toping:

$$\int \frac{dx}{(x+1)\sqrt{x^2+x-1}}.$$

Yechish. $t=\frac{1}{x+1}$yangi o'zgaruvchini kiritamiz:

$$\int \frac{dx}{(x+1)\sqrt{x^2+x-1}} = \left\{\begin{matrix} t=\frac{1}{x+1}; x=\frac{1}{t}-1; \\ dx=-\frac{1}{t^2}\ dt \end{matrix}\right\} =$$

$$= \int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t}-1\right)^2+\frac{1}{t}-1-1}} = -\int \frac{dt}{\sqrt{1-2t+t^2+t-2t^2}} =$$

$$= -\int \frac{dt}{\sqrt{1-t-t^2}} = -\int \frac{d\left(t+\frac{1}{2}\right)}{\sqrt{\frac{5}{4}-\left(t+\frac{1}{2}\right)^2}} = arccos\frac{t+\frac{1}{2}}{\frac{\sqrt{5}}{2}}+C =$$

$$= arccos\frac{2t+1}{\sqrt{5}}+C = arccos\frac{\frac{2}{x+1}+1}{\sqrt{5}}+C =$$

$$= arccos\frac{x+3}{\sqrt{5}(x+1)}+C.$$

Bo'laklab integrallash usuli

$$\int udv = uv - \int vdu$$

formulaga asoslanadi, bunda u va v – x ning integrallanuvchi funksiyalari.

5-misol. $\int xe^{-5x}dx$ ni toping.

Yechish. $u=x$va $dv=e^{-5x}dx$deb olamiz, u xolda

$$\int xe^{-5x}dx = \left\{\begin{matrix} u=x, du=dx, \\ dv=e^{-5x}dx, v=\int e^{-5x}dx=-\frac{1}{5}e^{-5x} \end{matrix}\right\} =$$

$$= -\frac{x}{5}e^{-5x} - \frac{1}{25}e^{-5x}+C.$$

$v$ni topishda integrallash doimiysini har doim nolga teng deb hisoblash mumkin.

6-misol. $\int arc\ tg\ xdx$ni toping.

Yechish. $u = arc\ tgx$deb olamiz, u holda

$$\int arc\ tgxdx = \begin{Bmatrix} u = arc\ tgx, du = \dfrac{dx}{1+x^2}, \\ dv = dx, v = \int dx = x \end{Bmatrix} =$$

$$= x \cdot arc\ tgx - \int \frac{xdx}{1+x^2} = x \cdot arc\ tgx - \frac{1}{2}\int \frac{d(1+x^2)}{1+x^2} =$$

$$= x \cdot arc\ tgx - \frac{1}{2} ln|1+x^2| + C.$$

7-misol. $\int (x^2+1)cosxdx$integralni toping.

Yechish. Bu misolda bo'laklab integrallash formulasini ikki marta qo'llashga to'g'ri keladi.

$$\int (x^2+1)cosxdx = \begin{Bmatrix} u = x^2+1, du = 2xdx; \\ dv = cosxdx, v = sinx \end{Bmatrix} =$$

$$= (x^2+1)sinx - 2(-xcosx + \int cosxdx) =$$

$$= (x^2+1)sinx + 2xcosx - 2sinx + C =$$

$$= 2xcos + (x^2-1)\ sinx + C.$$

8-misol. Aniqmas integralni hisoblang:

$$\int e^{ax} cos\beta xdx.$$

Yechish. Bu integralni ikki marta bo'laklab integrallaymiz.

$$\int e^{ax} cos\beta xdx = \begin{Bmatrix} u = e^{ax}, du = ae^{ax}dx; \\ dv = cos\beta xdx, v = \dfrac{1}{\beta} sin\beta x \end{Bmatrix} =$$

$$= \frac{1}{\beta} e^{ax} \cdot sin\beta x - \frac{\alpha}{\beta} \int e^{ax} sin\beta xdx =$$

$$= \begin{Bmatrix} u = e^{ax}, du = \alpha \cdot e^{ax}dx \\ dv = sin\beta xdx, v = -\dfrac{1}{\beta} cos\beta x \end{Bmatrix} =$$

$$= \frac{1}{\beta} e^{ax} \cdot sin\beta x - \frac{\alpha}{\beta}\left(-\frac{1}{\beta} e^{ax} cos\beta x + \frac{\alpha}{\beta} \int e^{ax} cos\beta xdx\right) =$$

$$= \frac{e^{ax}}{\beta^2}(\beta sin\beta x + \alpha cos\beta x) - \frac{\alpha^2}{\beta^2} \int e^{ax} cos\beta xdx.$$

Bunda

$$1 = \int e^{ax} cos\beta xdx$$

deb ushbu tenglikka ega bo'lamiz:

$$I = \frac{e^{ax}}{\beta^2}(\beta sin\beta x + \alpha cos\beta x) - \frac{\alpha^2}{\beta^2}\ I.$$

Bu tenglamani I ga nisbatan yechsak,

$$I = \int e^{\alpha x} \cos\beta x dx = \frac{e^{\alpha x}}{\alpha^2 + \beta^2}(\beta \sin\beta x + \alpha \cos\beta x) + C.$$

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