

TRIGONOMETRIK KO'RINISHDAGI INTEGRALLARNI INTEGRALLASH USULLARI

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Annotatsiya:

Ushbu maqolada trigonometrik ko'rinishdagi integrallarni integrallash usullari tadqiq qilingan. Trigonometrik funksiyalarni o'z ichiga olgan murakkab ifodalarni yechishda universal trigonometrik o'rinlashtirish, darajani pasaytirish va ko'paytmani yig'indiga keltirish kabi metodlarning samaradorligi tahlil etilgan. Maqolada keltirilgan uslubiy tavsiyalar va amaliy misollar oliy matematika kursida murakkab integrallarni hisoblash jarayonini soddalashtirishga xizmat qiladi.

Kalit so'zlar: Trigonometrik integrallar, darajani pasaytirish, o'zgaruvchini almashtirish, ratsional funksiyalar, integrallash algoritmlari.

Bu mavzuning dolzarbligi nafaqat sof matematik masalalarda, balki fizika (tebranishlar nazariyasi), muhandislik va iqtisodiy modellarni qurishda trigonometrik bog'lanishlarning keng qo'llanilishi bilan belgilanadi. Ko'p hollarda transsendent ko'rinishdagi integrallar hisoblashda qiyinchilik tug'diradi. Ushbu maqolaning maqsadi — trigonometrik integrallarni yechishning turli algoritmlarini tizimlashtirish va murakkab misollar orqali ularning eng samarali yo'llarini ko'rsatib berishdir.[1-2].

Ushbu metod integral ostida funksiyaning o'zi va uning hosilasi (differensial) mavjud bo'lganda qo'llaniladi.

Algoritm: Agar integral $R(\sin x) \cos x dx$ ko'rinishida bo'lsa, $u = \sin x$ deb olinadi. Agar $R(\cos x) \sin x dx$ bo'lsa, $u = \cos x$ belgilash kiritiladi.

Darajani pasaytirish usuli. Agar trigonometrik funksiyalar juft darajada kelsa, ularni chiziqli ko'rinishga keltirish uchun quyidagi ayniyatlardan foydalaniladi:

$$\sin^2 x = \frac{1 - \cos 2x}{2}; \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

Universal trigonometrik o'rinlashtirish. Ixtiyoriy ratsional $R(\sin x, \cos x)$ funksiyalar uchun $t = \tan \frac{x}{2}$ belgilash qo'llaniladi. Bu usulning afzalligi shundaki, u har qanday transsendent integralni ratsional kasrlarni integrallash masalasiga keltirib beradi.

Ko'paytmani yig'indiga keltirish. Argumentlari har xil bo'lgan ko'paytma ko'rinishidagi integrallarda quyidagi formulalar muhim:

$$1) \sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha - \beta) + \sin(\alpha + \beta)];$$

$$2) \sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)];$$

$$3) \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)].[2-4].$$

O'zgaruvchini almashtirish usuli. Misol: $\int \frac{\sin^{11}x}{\cos^{13}x} dx$

Tahlili: Bu integralda darajalar nisbatiga e'tibor berilsa, uni tangens funksiyasiga keltirish mumkin:

$$\int \frac{\sin^{11}x}{\cos^{13}x} \frac{1}{\cos^2x} dx = \int \operatorname{tg}^{11}x \frac{1}{\cos^2x} dx. \text{ Bu yerda } \operatorname{tg}x = u \text{ belgilash kiritsak, } du = \frac{dx}{\cos^2x} \text{ bo'ladi.}$$

$$\text{Yechimi: } \int u^{11} du = \frac{u^{12}}{12} + C = \frac{\operatorname{tg}^{12}x}{12} + C.$$

Xulosa: Darajalar farqi 2 ga teng bo'lgan sinus va kosingus nisbatlari har doim $u = \operatorname{tg}x$ belgilash orqali algebraik ko'phadga keltiriladi.[5-9].

Darajani pasaytirish usuli. Misol: $\int \cos^4x \sin^2x dx$

Tahlili: Juft darajali funksiyalar ko'paytmasini integrallashda darajani pasaytirish formulalaridan foydalaniladi:

$$\begin{aligned} \text{Yechimi: } \int \cos^2x(\sin x \cos x)^2 dx &= \int \frac{1+\cos 2x}{2} \frac{\sin^2x}{4} dx = \frac{1}{8} \int (\sin^2 2x + \sin^2 2x \cos 2x) dx = \\ &= \frac{1}{8} \left(\int \frac{1-\cos 4x}{2} dx + \int \sin^2 2x d\left(\frac{\sin 2x}{2}\right) \right) = \int \frac{1}{16} dx + \frac{1}{16} \int \cos 4x dx + \frac{1}{8} \int \sin^2 2x d\left(\frac{\sin 2x}{2}\right) = \frac{1}{16}x - \frac{\sin 4x}{64} + \frac{\sin^3 2x}{48} \\ &+ C. [10-15]. \end{aligned}$$

Universal trigonometrik o'rinashtirish. Misol: $\int \frac{dx}{3+5\cos x}$

Tahlili: Ratsional ko'rinishdagi ushbu integralni yechish uchun $t = \operatorname{tg}\frac{x}{2}$ belgilashidan foydalanamiz.

$$\text{Bunda } dx = \frac{2dt}{1+t^2} \text{ va } \cos x = \frac{1-t^2}{1+t^2}.$$

$$\text{Yechimi: } \int \frac{\frac{2dt}{1+t^2}}{3+5\left(\frac{1-t^2}{1+t^2}\right)} = \int \frac{2dt}{3+3t^2+5-5t^2} = \int \frac{2dt}{8-2t^2} = \int \frac{dt}{4-t^2} = \frac{1}{4} \ln \left| \frac{2+t}{2-t} \right| + C = \frac{1}{4} \ln \left| \frac{2+\operatorname{tg}\frac{x}{2}}{2-\operatorname{tg}\frac{x}{2}} \right| + C$$

Ko'paytmani yig'indiga keltirish usuli. Misol: $\int \sin 2x \cos 3x \sin 4x dx$

Tahlili: Uchta funksiya ko'paytmasini ketma-ket yig'indiga keltiramiz. Avval

$$\sin 2x \sin 4x \text{ qismini ko'rib chiqamiz: } \sin 2x \sin 4x = \frac{1}{2}(\cos 2x - \cos 6x).$$

$$\begin{aligned} \text{Yechimi: } \frac{1}{2} \int (\cos 2x - \cos 6x) \cos 3x dx &= \frac{1}{2} \int \cos 6x \cos 3x dx \text{ Formulani yana bir bor qo'llasak:} \\ \frac{1}{4} \int (\cos x + \cos 5x) dx - \frac{1}{4} \int (\cos 3x + \cos 9x) dx &\text{ Natija: } \frac{1}{4} \sin x + \frac{1}{20} \sin 5x - \frac{1}{12} \sin 3x - \frac{1}{36} \sin 9x + C. [16-} \\ &21]. \end{aligned}$$

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