PRODUCT CONTENT PLANNING MODEL

Sharofutdinov Iqboljon Osmonjon oglu
Senior Lecturer, Fergana State University, PhD
Email:Iqbol0766@gmail.com

Shavkatova Dilyorakhan Iqboljon kizi
Student at Fergana State University
Email:dildoradhavkatova@gmail.com

Abstract

In this article, the problem of product mix planning is mathematically modeled on the basis of linear programming. The goal of the model is to determine the optimal production plan that maximizes total profit under the conditions of limited resources of the enterprise (labor time, equipment capacity, raw material reserves and market demand). As a solution, an optimization approach based on simplex/HiGHS-type algorithms is used, which resources in the optimal plan are binding (binding), which are non-binding (reserve), as well as an economic interpretation is given through shadow price (marginal value) indicators of resources. The results show that even a product with a positive unit profit can fall out of the optimal portfolio due to the “cost” of the link resources; therefore, not only the margin, but also the return per unit of the link resource is a decisive factor in the formation of the assortment.

Keywords: Product composition; product mix; linear programming; simplex method; resource constraint; joint resource; shadow price; reduced cost; sensitivity analysis; optimization; operations research; production planning.

INTRODUCTION

In world practice, one of the most important tasks of production management is the optimal planning of the product mix. In conditions of increased competition, fluctuating raw material and energy prices, increased disruptions in supply chains, and seasonal/segmentary fluctuations in demand, the enterprise is forced to answer the question “how much of which product should be produced?” with a decision based on calculations, rather than intuitive ones. In such conditions, the issue of product mix planning is often characterized by several constraints at the same time (capacity, labor, equipment time, raw materials, logistics, demand limits), and their coordination requires an optimization approach.

Linear programming and its classical algorithms are widely used in global practice as a scientific and practical solution to this problem. The role of linear programming in production planning is especially associated with G. Dantzig's simplex algorithm, which gave a strong impetus to the development of applied operations research approaches in planning and management decision-making in large enterprises and systems.[1]

In global corporate practice, product mix models have not only remained an academic issue, but have also been used to formulate real management policies (marketing-production balance, assortment policy, capacity allocation, profitability improvement) as shown in numerous studies. In particular, there are practical applications of linear programming models that combine the elements of product

[1] https://www.stat.uchicago.edu/~lekheng/courses/309f11/top10/simplex.pdf

composition, sequencing, and blending in the analysis of corporate policy, which allow for a quantitative assessment of the economic results resulting from management decisions.[2]

RESULTS AND DISCUSSION

The purpose of analyzing the results of mathematical modeling of the problem of product composition planning is to determine, not only through intuitive decisions, but also through a quantitatively based optimization approach, which products are most optimal for an enterprise to produce in limited resources, to interpret the obtained optimal plan in terms of economic content, and to assess how the optimal decision changes (sensitivity and stability) when resources, prices, or technological standards change. Therefore, in the results and discussion section, management conclusions are drawn not only on the optimal plan itself, but also on the basis of tools such as “bottleneck” logic, dual prices (shadow price), reduced cost, and scenario analysis that led to the optimal plan. In the considered example, there are three products, the quantity produced from each of them is taken as a decision variable; the resources consist of labor time, equipment (power) time, and raw material reserves. The goal of the model is to maximize net profit, where the margin (net profit) from each unit of product is taken as , , respectively. Thus, the objective function is written as follows:$x_1, x_2, x_3 p_1 = 50 p_2 = 40 p_3 = 35$

max $Z = 50x_1 + 40x_2 + 35x_3$.

The restrictions are based on the principle that "the total expenditure on each resource should not exceed the available reserve"; that is, on labor

$$2x_1 + 1x_2 + 1.5x_3 \leq 100$$

by device time

$$1x_1 + 1.5x_2 + 1x_3 \leq 80$$

by raw material

$$3x_1 + 2x_2 + 2.5x_3 \leq 120$$

Additionally, market or production limits are given in the form of a demand constraint:

$0 \leq x_1 \leq 40, , .0 \leq x_2 \leq 600 \leq x_3 \leq 50$

This formulation is a classic example of practical product mix problems, and allows for a solution within the framework of linear programming; the important assumption here is that the resource consumption for the production of a unit of product is constant (linear) and the margin is also proportional to the volume of production (linear). It is this assumption that gives very favorable results in cases of planning production on the basis of "average normative"; if the technology has strong batch (setup), discrete capacity, or economies of scale, then at the next stage we move on to mixed-whole (MIP) or nonlinear models, but within the framework of this article, the linear model provides a sufficient basis for revealing the essence of the product mix problem.

The initial data of the model (resource consumption and reserves) are summarized in the table below; this table is the most important set of “input data” in practical terms, the incorrect selection of which may lead to the optimal solution not being suitable for real life. Therefore, usually, resource consumption coefficients are formed on the basis of a technological map, regulatory documents or real production monitoring (average consumption), and resource reserves are determined by the available capacity and reserves for the planned period (day/week/month).$a_{ji} b_j$

[2]https://pubsonline.informs.org/doi/10.1287/mnsc.24.13.1342

Table 1 Resource costs and resource reserves$a_{ji} b_j$

Resources / Products	**Product 1 ()x_1**	**Product 2 ()x_2**	**Product 3 ()x_3**	**Backupb_j**
Labor (hours)	2.0	1.0	1.5	100
Equipment time (hours)	1.0	1.5	1.0	80
Raw material (kg)	3.0	2.0	2.5	120

This table shows that products "consume" resources differently: for example, they require relatively more raw materials and labor (raw materials 3 kg, labor 2 hours), require more equipment time (1.5 hours), and consume 2.5 kg of raw materials, 1.5 hours of labor, and 1 hour of equipment. Now, it is precisely this "technological profile" and the combination of profit coefficients that form the optimal product mix. When the model is solved, the optimal plan turns out to be , , and the maximum profit is equal to . It is not enough to accept this solution as "numbers" alone; the central point of the result and discussion is that the mechanism that gave rise to this optimal plan must be interpreted in terms of resource constraints and economic content. First, we show the contribution of products to profit in the optimal plan: the profit contribution by , by , and by . So, 82–83% of the total 2320 profits are formed at the expense of the company, and the rest is accounted for by the company; it is not produced at all. This situation naturally raises the question "why did a product with positive profits fall to zero?" and illustrates one of the most important management lessons of product mix issues: the fact that a single unit of a product has a positive profit is not a sufficient condition for its inclusion in the optimal composition; the decisive factor is the relative efficiency of the product in terms of its bottleneck resources (fully occupied resources), that is, the return per bottleneck unit.$x_1 x_2 x_3 x_1^* = 8 x_2^* = 48 x_3^* = 0 Z^* = 2320 x_1 50 \cdot 8 = 400 x_2 40 \cdot 48 = 1920 x_3 35 \cdot 0 = 0 x_2 x_1 x_3$

Table 2 Optimal plan and profit contribution

Product	**Optimal amountx_i^***	**Unit profitp_i**	**Total profit contribution$p_i x_i$**
x_1	8	50	400
x_2	48	40	1920
x_3	0	35	0
Total			**2320**

To further interpret the optimal plan, we examine the resource utilization. If we put the optimal values into the constraints, the labor consumption is hours; this means that 64 hours out of 100 hours are spent, leaving 36 hours of spare. The equipment time consumption is hours, and exactly the available 80 hours of spare are fully used; that is, the slack in terms of equipment time is zero. The raw material consumption is kg, and the available 120 kg of raw material are also fully used; slack is zero. Thus, the resource utilization results show that in the optimal plan, two resources — equipment time and raw material — are fully used, while labor is partially used. This clearly demonstrates the concept of "bound resources": the optimal profit is limited by exactly the fully used resources, while labor does not play a limiting role. In mathematical terms, this situation can be expressed as follows using the slack (residual) variables:$x 2 \cdot 8 + 1 \cdot 48 + 1.5 \cdot 0 = 64 1 \cdot 8 + 1.5 \cdot 48 + 1 \cdot 0 = 80 3 \cdot 8 + 2 \cdot 48 + 2.5 \cdot 0 = 120$

$$s_1 = 100 - (2x_1 + x_2 + 1.5x_3) = 36$$
$$s_2 = 80 - (x_1 + 1.5x_2 + x_3) = 0$$
$$s_3 = 120 - (3x_1 + 2x_2 + 2.5x_3) = 0$$

Then, according to the idea of complementarity, a non-tied resource will have slack>0, while tied resources will have slack=0; in practical terms, this answers the question "will profit increase if the supply of this resource increases?"

Table 3 Resource consumption, backup and slack

Resource	Used	Backup	Slack (residue)	Employment (%)
Labor (hours)	64	100	36	64%
Equipment time (hours)	80	80	0	100%
Raw material (kg)	120	120	0	100%

Now, in the economic discussion of the result, dual (sensitivity) analysis tools play a very important role. The dual model of linear programming expresses the "intrinsic value" of resources in the product mix problem — that is, how much the increase in the resource stock by 1 unit increases the total profit. In the case of the primal problem and constraints, the dual problem usually has the form , satisfying the conditions , (there are equivalent options depending on the designation). The most important result for us from a practical point of view is the shadow prices, the content of which is . That is, if the -resource stock increases slightly, the optimal profit increases by approximately the coefficient (with the optimal base unchanged). According to the calculation results, the shadow price for labor is , for machine time is , for raw materials is . This result is fully consistent with slack analysis: there is a reserve in labor (slack 36), which means that it does not limit profit in the current regime and its marginal value is 0; Since machine time and raw materials are fully employed, their marginal cost is positive, meaning they are true finite resources.max $p^{\top}xAx \leq b$min $b^{\top}yA^{\top}y \geq py \geq 0y_j\Delta Z \approx y_j\Delta b_j j b_j Z^* y_j 0814$

If we turn to management decisions based on the results, the model first recommends restructuring the enterprise strategy around "bottleneck resources". If the enterprise wants to increase profits in conditions where raw materials and equipment are fully occupied, the following ways are most logical: (i) expand the raw material reserve (new supplier, long-term contract, reserve policy), (ii) reduce raw material consumption (reduce standards, reduce waste, rework), (iii) increase equipment time (additional shifts, maintenance, reduce downtime), (iv) make equipment time "effective" (increase OEE, reduce setup time), (v) change the product design or technological process in such a way that the consumption of the link resources is reduced and more profit is obtained from each unit of the resource. Shadow price analysis also gives the economic priority of these areas: the largest in raw materials; This means that increasing the stock of raw materials or reducing the consumption of raw materials (effectively increasing "b") has the largest marginal effect on profit. It is on equipment, which is also significant. Since it is on labor, increasing labor is not profitable; this, for example, can be used to test and refute the hypothesis of the enterprise that "there is a shortage of workers, therefore profits are low": according to the model, there is no shortage of workers (labor hours) in the current conditions, but rather the resources are different.$y = 14y = 8y = 0$

Another important part of the discussion is the stability of the model result. In real life, profit coefficients and resource costs can be variable: energy prices, raw material prices, labor costs, logistics costs, as well as changes in the technological regime change the margin. Therefore, it is important to know the "stable zone" of the optimal plan: at what range the plan does not change, even if there is a change, and at what point the plan changes structurally. In our example, the plan did not change even if it increased from 35 to 40, which indicates a certain stability of the plan relative to ; and around , a structural change began. This means that small fluctuations in prices and costs do not require constant rewriting of the enterprise plan, but during significant jumps (for example, new market prices, sharp cost reductions), the plan must be reoptimized. Similarly, resource reserves also change under the influence of seasonal or organizational factors (raw material delays, equipment breakdowns, unplanned downtime); Through shadow price analysis, it is possible to predict in advance which resource change will "hit" profits the most: a decrease in raw material and equipment inventory significantly reduces profits, while a decrease in labor may not have any effect to a certain extent under current conditions (due to the presence of slack 36).$p_i a_{ji} p_3 p_3 p_3 \approx 43 b_j$

Based on the results, strategic conclusions are also drawn regarding the product portfolio. In the optimal plan, it becomes the main product, because it gives the highest total profit in combination with the associated resources. However, this does not mean that “it is necessary to produce only”; it is also in the portfolio, because it, together with the geometry of the constraints, fills the associated resources and forms a highly profitable corner point. is excluded for now, but this does not mean that it is “a completely unnecessary product”; it may be necessary for marketing, branding or diversification. In this case, if the enterprise is forced to produce a minimum amount of “mandatory assortment”, then the minimum limit is introduced into the model and the optimal plan is recalculated; as a result, the total profit may decrease, and this decrease is explained by reduced cost and associated resources. That is, the model allows you to coordinate the product mix issue not only with “profit”, but also with the assortment obligations of the enterprise. Also, if the product is planned to be developed, the model asks the question “how will it enter the portfolio if we develop it?” gives a quantitative answer to the question: increase (increase price or decrease cost) or reduce costs (improve technology). Here the choice is in the hands of the manager: for example, increasing the price may affect market demand, while reducing cost requires investment; if a change in technology reduces the cost of equipment or raw materials, the efficiency of the link in terms of resources increases and the probability of entering the portfolio increases.$x_2 x_2 x_1 x_2 x_3 x_3 x_3 \geq \ell_3 x_3 p_3 a_{j3} x_3$

CONCLUSION

Conclusion. The results of this study clearly confirmed that the product mix model is one of the most effective mathematical modeling approaches that allows an enterprise to make a scientifically based choice of production decisions in conditions of limited resources. Through a model built on the basis of linear programming, the net profit of each product, resource consumption standards and available reserves are combined into a single system, and an optimal plan that maximizes profit is determined. Calculations showed that in the optimal plan, resources do not have the same impact: some resources (link resources) are fully used, limiting the total profit, while others, having reserves, do not have a marginal impact on profit. This situation serves to correctly determine investment and operational priorities in enterprise management: to increase profit, first of all, it is necessary to expand link resources or reduce their consumption, while increasing a non-link resource may not give the expected result.

The model results also prove that the profitability of a product (positive unit profit) does not guarantee its optimal production: if a product consumes a unit of resources relatively “expensively”, it will crowd out the opportunity for other products to make a profit and will fall out of the optimal composition. This conclusion is important for assortment policy: when forming a product portfolio, an enterprise should rely not only on the margin, but also on the criteria of return per unit of resource (profit density) and lost opportunity cost (opportunity cost). Sensitivity (shadow price) analysis allows for a quantitative assessment of the internal economic value of resources and helps to predict the impact of increasing resource reserves on profit; reduced cost analysis determines the threshold of the minimum economic improvement (price increase, cost reduction, or resource consumption reduction) necessary for a non-produced product to enter the portfolio.

In general, the product mix planning model, as a "core" instrument of an enterprise planning system, provides the following practical results:

(1) increase profits by selecting optimal production volumes;

(2) identify scarce resources and develop specific recommendations for their management;

(3) optimize the assortment in an economically sound manner;

(4) Assess the sustainability of the plan and quickly re-plan when prices, costs, and resource reserves change.

Therefore, when implementing this approach in real enterprise conditions, it is recommended that the main focus be on reliably forming the database (margin, regulatory costs, capacity and demand constraints), and at the next stage, gradually integrating factors such as batching, minimum assortment, logistics and demand uncertainty into the model.

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